Trigonometrie

$$\sin^2 x + \cos^2 x = 1 \qquad \tan x = \frac{\sin x}{\cos x}$$

$$\cos^2 x = \frac{1}{1+\tan^2 x} \qquad \sin^2 x = \frac{\tan^2 x}{1+\tan^2 x} \qquad 1+\tan^2 x = \frac{1}{\cos^2 x}$$

$$\sin(-x) = -\sin(x) \qquad \sin(\pi - x) = \sin(x) \qquad \sin(\pi + x) = -\sin(x)$$
$$\cos(-x) = \cos(x) \qquad \cos(\pi - x) = -\cos(x) \qquad \cos(\pi + x) = -\cos(x)$$
$$\tan(-x) = -\tan(x) \qquad \tan(\pi - x) = -\tan(x) \qquad \tan(\pi + x) = \tan(x)$$

$$\sin\left(\tfrac{\pi}{2} - x\right) = \cos(x) \qquad \sin\left(\tfrac{\pi}{2} + x\right) = \cos(x)$$
$$\cos\left(\tfrac{\pi}{2} - x\right) = \sin(x) \qquad \cos\left(\tfrac{\pi}{2} + x\right) = -\sin(x)$$
$$\tan\left(\tfrac{\pi}{2} - x\right) = \cot(x) \qquad \tan\left(\tfrac{\pi}{2} + x\right) = -\cot(x)$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y \qquad \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y \qquad \tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\sin 2x = 2 \sin x \cos x \qquad 2\cos^2 x = 1 + \cos 2x$$
$$\cos 2x = \cos^2 x - \sin^2 x \qquad 2\sin^2 x = 1 - \cos 2x$$

$$\sin 2x = \frac{2\tan x}{1+\tan^2 x} \qquad \cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x} \qquad \tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\sin 3x = 3\sin x - 4\sin^3 x \qquad \cos 3x = -3\cos x + 4\cos^3 x$$

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) \qquad \tan x + \tan y = \frac{\sin(x+y)}{\cos x \cos y}$$
$$\sin x - \sin y = 2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right) \qquad \tan x - \tan y = \frac{\sin(x-y)}{\cos x \cos y}$$
$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\sin x \cos y = \tfrac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$$
$$\cos x \cos y = \tfrac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$$
$$\sin x \sin y = \tfrac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

Elementare Ableitungen

Funktionsklasse	**Funktion $f(x)$**	**Ableitung $f(x)'$**
konstante Funktion	c	0
Potenzfunktion	x^n mit $n \in R$	$n \cdot x^{n-1}$
Wurzelfunktion	$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
Trigonometrische Funktionen	$\sin(x)$ $\cos(x)$ $\tan(x)$ $\cot(x)$	$\cos(x)$ $-\sin(x)$ $\frac{1}{\cos^2(x)}$ $\frac{1}{-\sin^2(x)}$
Arcusfunktionen	$\arcsin(x)$ $\arccos(x)$ $\arctan(x)$ $\operatorname{arccot}(x)$	$\frac{1}{\sqrt{1-x^2}}$ $-\frac{1}{\sqrt{1-x^2}}$ $\frac{1}{1+x^2}$ $-\frac{1}{1+x^2}$
Exponentialfunktionen	e^x a^x	e^x $\ln(a) \cdot a^x$
Logarithmusfunktionen	$\ln(x)$ $\log_a(x)$	$\frac{1}{x}$ $\frac{1}{\ln(a) \cdot x}$

Elementare Stammfunktionen

1) $\displaystyle\int x^p\,dx = \frac{x^{p+1}}{p+1} + C \qquad (p \neq -1)$

2) $\displaystyle\int \frac{dx}{x} = \ln|x| + C$

3) $\displaystyle\int e^x\,dx = e^x + C$

4) $\displaystyle\int a^x\,dx = \frac{a^x}{\ln a} + C \qquad (a > 0, a \neq 1)$

5) $\displaystyle\int \sin x\,dx = -\cos x + C$

6) $\displaystyle\int \cos x\,dx = \sin x + C$

7) $\displaystyle\int \tan x\,dx = -\ln|\cos x| + C$

8) $\displaystyle\int \cot x\,dx = \ln|\sin x| + C$

9) $\displaystyle\int \frac{dx}{\cos^2 x} = \int \left(1 + \tan^2 x\right) dx = \tan x + C$

10) $\displaystyle\int \frac{dx}{\sin^2 x} = \int \left(1 + \cot^2 x\right) dx = -\cot x + C$

11) $\displaystyle\int \sinh x\,dx = \int \frac{e^x - e^{-x}}{2}\,dx = \cosh x + C$

12) $\displaystyle\int \cosh x\,dx = \int \frac{e^x + e^{-x}}{2}\,dx = \sinh x + C$

13) $\displaystyle\int \tanh x\,dx = \int \frac{e^{2x} - 1}{e^{2x} + 1}\,dx = \ln(\cosh x) + C$

14) $\displaystyle\int \coth x\,dx = \int \frac{e^{2x} + 1}{e^{2x} - 1}\,dx = \ln|\sinh x| + C$

15) $\displaystyle\int \frac{dx}{\cosh^2 x} = \int \left(1 - \tanh^2 x\right) dx = \tanh x + C$

16) $\displaystyle\int \frac{dx}{\sinh^2 x} = \int \left(-1 + \coth^2 x\right) dx = -\coth x + C$

17) $\displaystyle\int \frac{dx}{1 + x^2} = \begin{cases} \arctan x + C_1 \\ -\operatorname{arccot} x + C_2 \end{cases} \qquad \arctan x = \frac{\pi}{2} - \operatorname{arccot} x$

18) $\displaystyle\int \frac{dx}{1 - x^2} = \left\{ \begin{array}{lcll} \operatorname{Artanh} x + C_1 & : & |x| < 1 & \operatorname{Artanh} x = \frac{1}{2}\ln\frac{1+x}{1-x} \\ \operatorname{Arcoth} x + C_2 & : & |x| > 1 & \operatorname{Arcoth} x = \frac{1}{2}\ln\frac{x+1}{x-1} \end{array} \right\} \frac{1}{2}\,\ln\left|\frac{x+1}{x-1}\right|$