

3. Aufgabe:

Lösen Sie die Matrixgleichung $A \cdot X = B$ mit:

$$\begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

Lösung:

$$\begin{array}{l} (A \mid B) \\ (A^{-1} \cdot A \mid A^{-1} \cdot B) \\ (E \mid X) \end{array} \quad \begin{array}{l} \text{”gedanklich” beide Seiten Mit } A^{-1} \text{ multiplizieren} \\ \text{mit } X = A^{-1} \cdot B \\ \text{umformen von } (A \mid B), \text{ so dass links } E \text{ steht.} \end{array}$$

$$\left(\begin{array}{cccc|cccc} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 1 & 0 & -1 & 1 & -1 \\ 1 & 0 & 0 & -1 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{array} \right) \xrightarrow{z_1+z_3 \rightarrow z_3} \left(\begin{array}{cccc|cccc} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & -1 & 2 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{array} \right) \xrightarrow{z_3-z_2 \rightarrow z_2}$$

$$\left(\begin{array}{cccc|cccc} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & -1 & 2 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 2 & -1 & 2 \end{array} \right) \xrightarrow{z_4-z_3 \rightarrow z_3} \left(\begin{array}{cccc|cccc} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & -1 & 2 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & -2 & 2 & 0 & 3 \end{array} \right) \xrightarrow{\begin{array}{l} z_2-z_4 \rightarrow z_2 \\ z_3+z_4 \rightarrow z_3 \end{array}}$$

$$\left(\begin{array}{cccc|cccc} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 2 & -3 & 1 & -4 \\ 0 & 0 & 1 & 0 & 0 & 2 & -1 & 2 \\ 0 & 0 & 0 & 1 & -2 & 2 & 0 & 3 \end{array} \right) \xrightarrow{\begin{array}{l} z_1-z_3 \rightarrow z_1 \\ z_2+z_3 \rightarrow z_2 \end{array}} \left(\begin{array}{cccc|cccc} -1 & 0 & 0 & 0 & 1 & -2 & 0 & -2 \\ 0 & 1 & 0 & 0 & 2 & -1 & 0 & -2 \\ 0 & 0 & 1 & 0 & 0 & 2 & -1 & 2 \\ 0 & 0 & 0 & 1 & -2 & 2 & 0 & 3 \end{array} \right) \xrightarrow{-z_1 \rightarrow z_1}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & 2 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 & -1 & 0 & -2 \\ 0 & 0 & 1 & 0 & 0 & 2 & -1 & 2 \\ 0 & 0 & 0 & 1 & -2 & 2 & 0 & 3 \end{array} \right) \Rightarrow X = \begin{pmatrix} -1 & 2 & 0 & 2 \\ 2 & -1 & 0 & -2 \\ 0 & 2 & -1 & 2 \\ -2 & 2 & 0 & 3 \end{pmatrix}$$